Energy correction schemes in space and time

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partly joint work with M. Melenk (TU Wien), H. Egger (TU Darmstadt), U. Rüde (FAU) and with F. Benesch, T. Horger, M. Huber, T. Köppl, C. Waluga (TU München)

In this talk, we address the role of the energy within numerical schemes for partial differential equations (pdes). We focus on two different aspects: stability of time integration and a priori estimates for spatial discretizations.

Lagrange multiplier based space discretizations for time dependent constrained pdes result quite often in DAE systems of index three. Thus typically spurious oscillations can be observed in case of standard time integrations schemes. Here we show how simple variational crimes based on suitable quadrature rules can reduce the index from three to one and yield stable energy preserving time integration schemes. As model example we consider a dynamic contact problem with Coulomb friction and illustrate numerically the influence of a local mass redistribution.

In the second part, we focus on stationary elliptic problems where the solution is globally non-smooth. Typical examples are domains with re-entrant corners, heterogeneous coefficients and right hand sides being not in the H^1-dual space. These situations occur quite often in the mathematical modelling of multi-physics applications. Standard remedies to handle these type of problems are graded meshes or enrichment, both techniques result in extra implementational work and computational cost. In the case of point sources, we provide optimality results and show that globally no pollution occurs. Although the solution is globally not in H^1, we observe on a sequence of uniformly refined meshes optimal L^2-norm a priori convergence on subdomains excluding the point sources. In the case of heterogeneous coefficients or re-entrant corners, the situation is more complex. Then the well-known pollution effect is observed, and the convergence can be extremely poor. This also holds true not only for the L^2 norm on subdomains excluding the cross-points and corners but also for other quantities of interests such as the stress intensity factors or eigenvalues. Here we introduce a purely local energy correction function and modify locally the bilinear form. Numerical examples in 2D illustrate that we can recover full optimality in case of uniform meshes for re-entrant corners, heterogeneous coefficients and linear elasticity. As quantities of interest we select the convergence of eigenvalues, the flux across an interface and the stress intensity factor.

[1] C. Hager, B. Wohlmuth: *Analysis of a space-time discretization for dynamic elasticity problems based on mass-free surface elements*, 2009, SIAM J. Numer. Anal., 47, 1863-1885

[2] H. Egger, U. Rüde, B. Wohlmuth: *Energy-corrected finite element methods for corner singularities*, to appear in SIAM J. Numer. Anal.

[3] T. Köppl, B. Wohlmuth: *Optimal a priori error estimates for an elliptic problem with Dirac right-hand side*, submitted 2013

[4] U. Rüde, C. Waluga, B. Wohlmuth: *A multi-level algorithm for energy-corrected finite element methods for corner singularities*, submitted 2013